

# Faraday Rotation in Global Accretion Disk Simulations: Implications for Sgr A\*

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## ABSTRACT

We calculate Faraday rotation in global axisymmetric magnetohydrodynamic simulations of geometrically thick accretion flows. These calculations are motivated by the measured rotation measure (RM) of  $\approx -6 \times 10^5 \text{ rad m}^{-2}$  from Sgr A\* in the Galactic center, which appears to have been stable over the past  $\approx 7$  years. In our numerical simulations, the quasi-steady state structure of the accretion flow, and the RM it produces, depends on the initial magnetic field threading the accreting material. In spite of this dependence, we can draw several robust conclusions about Faraday rotation produced by geometrically thick accretion disks: i) the time averaged RM does not depend that sensitively on the viewing angle through the accretion flow, but the stability of the RM can. Equatorial viewing angles show significant variability in RM (including sign reversals), while polar viewing angles are relatively stable if there is a large scale magnetic field threading the disk at large radii. ii) Most of the RM is produced at small radii for polar viewing angles while all radii contribute significantly near the midplane of the disk. Our simulations confirm previous analytic arguments that the accretion rate onto Sgr A\* must satisfy  $\dot{M}_{\text{in}} \ll \dot{M}_{\text{Bondi}} \sim 10^{-5} M_{\odot} \text{ yr}^{-1}$  in order to not over-produce the measured RM. We argue that the steady RM  $\approx -6 \times 10^5 \text{ rad m}^{-2}$  from Sgr A\* has two plausible explanations: 1) it is produced at  $\sim 100$  Schwarzschild radii, requires  $\dot{M}_{\text{in}} \approx 3 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ , and we view the flow at an angle of  $\sim 30^\circ$  relative to the rotation axis of the disk; in our simulations, the variation in RM across a finite-sized source is sufficient to depolarize the emission below  $\approx 100$  GHz, consistent with observations. 2) Alternatively, the RM may be produced in the relatively spherical inflowing plasma near the circularization radius at  $\sim 10^3 - 10^4$  Schwarzschild radii, with the magnetic field perhaps amplified by the magnetothermal instability. Time variability studies of the RM can distinguish between these two possibilities.

*Subject headings:* accretion, accretion disks – MHD – Galaxy: center

## 1. Introduction

Observations of stellar orbits in the Galactic center indicate the presence of a  $\approx 4 \times 10^6 M_{\odot}$  black hole at the center of the Galaxy (e.g., Schödel et al. 2002; Ghez et al. 2005). Numeri-

cal modeling of stellar outflows and X-ray observations of the diffuse thermal plasma in the central parsec show that winds from massive stars provide a mass supply of  $\dot{M}_{\text{Bondi}} \sim 10^{-6} - 10^{-5} M_{\odot} \text{ yr}^{-1}$  at  $\approx 10^5$  Schwarzschild radii, which is approximately the gravitational sphere of influence of

the black hole (Baganoff et al. 2003; Quataert 2004; Cuadra et al. 2006). The observed bolometric luminosity of the electromagnetic source Sgr A\* coincident with the black hole is, however, 5 orders of magnitude smaller than would be expected if all of this available gas were accreted with a radiative efficiency of 10%. As a result, the inflowing plasma must have a very low radiative efficiency (e.g., Narayan & Yi 1995) and/or very little of the gas supplied at large radii actually makes it down to the black hole (e.g., Blandford & Begelman 1999). Global MHD simulations of radiatively inefficient accretion flows show significant mass-loss consistent with the latter possibility (Stone & Pringle 2001, hereafter SP01; Hawley, Balbus, & Stone 2001). In addition, local numerical simulations of electron heating in collisionless accretion flows find that the electrons receive a significant fraction of the gravitational potential energy released in the accretion flow, requiring  $\dot{M} \ll \dot{M}_{\text{Bondi}}$  to explain the low luminosity of Sgr A\* (Sharma et al. 2007).

An important observational diagnostic of the physical conditions in the accreting plasma is polarization measurements. Linear polarization from Sgr A\* was first detected by Aitken et al. (2000) at sub-millimeter wavelengths; they found that the polarization fraction increases rapidly from 150 to 400 GHz. Linear polarization is not detected at lower frequencies, although circular polarization is (e.g., Bower et al. 1999, 2002). The decrease in the linear polarization fraction at low frequencies may be due to depolarization by differential Faraday rotation across the source (e.g., Bower et al. 1999). In addition, the emission at low frequencies is optically thick, which suppresses the linear polarization if the particle distribution function is roughly thermal (e.g., Goldston et al. 2005).

The detection of linearly polarized radiation at mm & sub-mm wavelengths rules out high accretion rate models in which  $\dot{M} \sim \dot{M}_{\text{Bondi}}$  near the black hole (Agol 2000; Quataert & Gruzinov 2000; Melia et al. 2000). Analytic estimates show that such models would produce a rotation measure (RM) much larger than observed—so large, in fact, that all of the mm emission would be depolarized by Faraday rotation. In the past five years, higher resolution constraints on the linear polarization of Sgr A\* have generally confirmed the Aitken et al. results (Bower et al. 2003;

Marrone et al. 2006; Macquart et al. 2006). Marrone et al. (2007) performed the first measurement of the polarization of Sgr A\* with sufficient sensitivity to determine the Faraday rotation in single-epoch observations, free from ambiguities introduced by possible time variation in the intrinsic polarization angle. They found  $RM \approx -6 \times 10^5$  rad m<sup>-2</sup> using observations at both 227 and 343 GHz. The observed RM appears to be relatively constant over the 2 years of high sensitivity SMA observations (Marrone, private communication). Comparison with previous observations suggests that the RM towards Sgr A\* has not changed sign over the past  $\approx 7$  years.

Propagation through the interstellar medium cannot account for the large RM observed towards Sgr A\*. In addition, X-ray observations of the thermal plasma in the central parsec of the Galactic center shows that the electron number density is  $\approx 100$  cm<sup>-3</sup> and the temperature is  $\approx 2$  keV at a distance of  $\approx 1'' \approx 0.04\text{pc} \approx 10^5 r_g$  from the black hole (Baganoff et al. 2003; Quataert 2004), where  $r_g$  is the Schwarzschild radius. If the magnetic field on this scale were uniform, purely radial, and in equipartition with the thermal pressure,  $B \approx 2$  mG and  $RM \approx 5000$  rad m<sup>-2</sup>, two orders of magnitude smaller than what is observed. This shows that the observed RM must be produced at distances  $\ll 10^5 r_g$  from the black hole, providing an important diagnostic of the physics of accretion onto Sgr A\*. In view of the uniqueness of this probe, and the importance of Sgr A\* as the paradigm for radiatively inefficient accretion flows, a more detailed study of the constraints imposed by the observed Faraday rotation is warranted.

In this paper we carry out two-dimensional (axisymmetric) MHD simulations of accretion onto a central point mass and study Faraday rotation through the resulting turbulent accretion flow. This significantly improves on previous analytic estimates of the Faraday rotation in radiatively inefficient accretion flows, which relied on simplifying assumptions such as power-law density profiles, uniform steady magnetic fields, etc. One caveat about our assumption of axisymmetry is that, by the anti-dynamo theorem, the magnetic field must eventually decay. However, the similarity of previous 2-D and 3-D simulations in the quasi-steady turbulent state, that lasts for hundreds of orbital periods at small radii

(Hawley, Balbus, & Stone 2001), gives us some confidence that our results will hold in 3-D.

The remainder of this paper is organized as follows. Our numerical techniques and methods for calculating RM are described in §2; we also briefly summarize the analytic scalings for RM in radiatively inefficient accretion flows (§2.3). In §3 we discuss our results on Faraday rotation in MHD simulations. We focus on a fiducial model, but discuss the sensitivity of our results to the initial magnetic field configuration in the simulation, numerical resolution, and the size of the computational domain. In §4 we summarize our results and discuss several scenarios that can account for the observed RM from Sgr A\*.

## 2. Methods and Analytic Scalings

### 2.1. Numerical Simulations

Unless specified otherwise, we use a numerical set up similar to that of SP01. We use the ZEUS-2D MHD code (Stone & Norman 1992a,b) with spherical  $(r, \theta)$  coordinates to solve the equations of ideal MHD under the assumption of axisymmetry. The gravitational potential is the pseudo-Newtonian potential of Paczynski & Wiita (1980), namely  $\Phi = -GM/(r - r_g)$ , where  $r_g = 2GM/c^2$ . The energy equation is adiabatic with the exception of heating by shock-capturing artificial viscosity (which is a rather minor source of heating). We do not use an explicit resistivity to capture some of the energy lost to grid-scale averaging as it does not appear to significantly affect the dynamics of the flow (e.g., see SP01).

The initial condition is a geometrically thick, weakly magnetized, rotationally and pressure supported constant angular momentum torus. The density in the initial torus is given by equation (6) of Stone, Pringle, & Begelman (1999) which is based on Papaloizou & Pringle (1984). The peak density of the torus is 1 in code units. The torus is truncated when the density falls below 0.01. It is surrounded by a constant density corona ( $\rho = 10^{-4}$ ); there is a density and temperature jump at the torus-corona boundary.

The torus is specified by the radius of the initial density maximum,  $r_0$ , which we take to be  $r_0 = 40r_g$ ,  $100r_g$ , or  $200r_g$ . The gravitational sphere of influence of Sgr A\* on the surrounding hot plasma is  $\sim 10^5 r_g$ . Three-dimensional nu-

merical simulations of accretion from stellar winds imply a circularization radius of  $\sim 10^3 - 10^4 r_g$  (Cuadra et al. 2006). Ideally we would thus like to simulate an accretion disk which is  $\sim 10^4 - 10^5 r_g$  in radial extent. This is, however, infeasible at the present time. Instead, our goal is to understand the generic properties of Faraday rotation through the steady state accretion flow structure that develops at radii  $\lesssim r_0$ .

Our grid is logarithmic with a higher density of points at small radii and in the midplane. The radial grid is spaced such that the number of grid points from  $r_{\min}$  to  $(r_{\min} r_{\max})^{1/2}$  is roughly the same as the number of grid points from  $(r_{\min} r_{\max})^{1/2}$  to  $r_{\max}$ , where  $r_{\min}$  and  $r_{\max}$  are the inner and outer radial boundaries. Three different resolutions,  $60 \times 44$ ,  $120 \times 88$ , and  $240 \times 176$ , are used. We use outflow boundary conditions and set  $B_r B_\phi \leq 0$  at  $r_{\min} = 2r_g$ .

We consider two different initial magnetic field configurations in the torus: configuration A in which the field lies along the constant density contours (Fig. 1) and is derived from the vector potential  $A_\phi = \rho^2/\beta_0$  with  $\beta_0 = 200$  (the same as runs B & F of SP01), and configuration B which has a net radial field in the equatorial plane (see Fig. 8 & the Appendix). We find that the results of our Faraday rotation calculations remain sensitive to the initial field configuration even in the turbulent state. This is discussed in more detail in §3.2 and §4.

The initial weakly magnetized torus is unstable to the magnetorotational instability (MRI; Balbus & Hawley 1991), which rapidly amplifies the magnetic field leading to MHD turbulence and efficient angular momentum transport. After an initial amplification stage, a quasi-steady turbulent accretion flow is established. However, at late times, due to the impossibility of a self sustained dynamo in 2D, the turbulent motions die away. We carry out the RM calculations in the quasi-steady turbulent state which lasts for hundreds of orbital periods at small radii.

### 2.2. Rotation Measure

Due to the different index of refraction for left and right circularly polarized radiation, the polarization angle of linearly polarized light is rotated by an angle of  $\lambda^2 \text{RM}$  on passing through

a non-relativistic magnetized plasma, where  $RM = (e^3/2\pi m_e^2 c^4) \int n \mathbf{B} \cdot d\mathbf{l}$  is the rotation measure,  $e$  and  $m_e$  are the electron charge and mass,  $c$  is the velocity of light in vacuum,  $n$  is the number density of electrons,  $\mathbf{B}$  is the magnetic field, and  $d\mathbf{l}$  is the differential length along the line of sight. For a relativistic plasma, RM is suppressed by a factor of  $\approx \ln(\theta)/\theta^2$  (Quataert & Gruzinov 2000), where  $\theta = kT_e/m_e c^2$ . In addition, propagation through a relativistic plasma can lead to conversion between linear and circular polarization (e.g., Ruszkowski & Begelman 2002; Ballantyne, Ozel, & Psaltis 2007). We focus on non-relativistic plasmas throughout this paper.

We calculate RM along different lines of sight through the simulation domain in the quasi-steady turbulent state. The MHD variables ( $\rho$ ,  $B$ , etc.) are output 40 times per orbital period at the density maximum  $r_0$ ; these variables are then used to calculate RM for different viewing angles. For Figure 7 we used output 160 times per orbital period to guarantee that the short timescale variability in RM was captured. Our results are, however, largely insensitive to the exact time sampling used.

The rotation measure in arbitrary code units is rescaled as follows

$$RM_{\text{code}} \equiv \left( \frac{\langle \dot{M}_{\text{code}} \rangle}{0.002} \right)^{-3/2} \times \int_{r_{\text{in}}}^{r_{\text{out}}} \rho \mathbf{B} \cdot d\mathbf{l} \quad (1)$$

where  $\langle \dot{M}_{\text{code}} \rangle$  is the average mass accretion rate at  $r_{\text{min}} = 2r_g$  during the turbulent steady state, and  $r_{\text{in}}$  and  $r_{\text{out}}$  are the inner and outer radii used in calculating the rotation measure. Numerical simulations with different resolution,  $r_0$ , and initial field configurations have different values of  $\langle \dot{M}_{\text{code}} \rangle$ . Equation (1) effectively re-scales all of the different simulations to have the same time averaged accretion rate, allowing a more useful comparison of the different simulations. The conversion of  $RM_{\text{code}}$  to a physical rotation measure (in  $\text{rad m}^{-2}$ ) is then given by

$$RM = 1.5 \times 10^{10} \left( \frac{\dot{M}_{\text{in}}}{10^{-5} M_{\odot} \text{yr}^{-1}} \right)^{3/2} \left( \frac{M_{\text{BH}}}{4 \times 10^6 M_{\odot}} \right)^{-2} \left( \frac{RM_{\text{code}}}{10^{-4}} \right) \quad (2)$$

where  $M_{\text{BH}}$  is the black hole mass and  $\dot{M}_{\text{in}}$  is the actual mass accretion rate through the horizon of the black hole (i.e., at  $r_{\text{min}} = 2r_g$ ).

Physically  $r_{\text{in}}$  in equation (1) corresponds to the radius where the electrons become non-relativistic, since there is little Faraday rotation from smaller radii. We choose  $r_{\text{in}} = 10r_g$  unless specified otherwise. The true  $r_{\text{in}}$  in Sgr A\* is probably somewhat larger,  $r_{\text{in}} \sim 100r_g$  (e.g., Sharma et al. 2007), but we are restricted to smaller values of  $r_{\text{in}}$  for numerical reasons. We discuss the scalings of our results to larger  $r_{\text{in}}$  in §4. The outer radius  $r_{\text{out}}$  is chosen so that the density structure of the accretion flow is not affected by the initial conditions. We choose  $r_{\text{out}} = 30r_g$  for  $r_0 = 40r_g$ ,  $r_{\text{out}} = 60r_g$  for  $r_0 = 100r_g$ , and  $r_{\text{out}} = 120r_g$  for  $r_0 = 200r_g$ .

We calculate the RM in two ways in the simulations: along individual lines of sight ( $\theta = \text{constant}$  in spherical coordinates) and in a cylindrical “beam” of diameter  $D_{\text{beam}}$ . Physically, the relevant beam size is expected to be the size of the synchrotron source which is itself a function of frequency, with  $D_{\text{beam}} \approx 10r_g(\nu/100 \text{ GHz})^{-1}$  for Sgr A\* (Bower et al. 2004; Shen et al. 2005; Loeb & Waxman 2007). We take  $D_{\text{beam}} = 10r_g$  but find that our results are not sensitive to a factor of  $\sim 2$  variation in this choice.

For an individual line of sight we evaluate the integral in equation (1) by adding up the contribution of each  $\theta = \text{constant}$  grid point that lies between  $r_{\text{in}}$  and  $r_{\text{out}}$ . For a beam of width  $D_{\text{beam}}$  all grid points lying within the beam contribute to the RM integral. This effectively assumes that the source has a constant surface brightness across the synchrotron photosphere. Since the size of the grid cells is not uniform, we calculate the beam averaged RM by calculating the average value of  $nB_{\parallel}$  within the beam, weighing the contribution of each grid cell by its area, and multiplying by  $r_{\text{out}} - r_{\text{in}}$  (the radial length of the region contributing to the calculated RM). Using the RM for each time-slice in the turbulent state we calculate the time averaged RM and the standard deviation in time, for both individual lines of sight and beams. The standard deviation in time provides a useful measure of the time variability of the RM. We also calculate the spatial variation in RM within the beam of width  $D_{\text{beam}}$ ; this can be used to quantify the effects of beam depolarization.

### 2.3. Analytic Scalings

To provide some context for the numerical simulations that follow, we briefly review the an-

alytic expectations for RM in radiatively inefficient accretion flows. For a flow with a density profile of  $n \propto r^{-3/2+p}$  and a virial temperature  $T \propto r^{-1}$ , the equipartition magnetic field strength should vary as  $B \propto (nT)^{1/2} \propto r^{-5/4+p/2}$ . Thus the local contribution of a given decade in radius to the rotation measure is given by  $dRM/d\ln r \propto rnB \propto r^{-7/4+3p/2}$ . For Bondi accretion,  $p = 0$  and  $dRM/d\ln r \propto r^{-7/4}$ ; RM is thus dominated by small radii. For Sgr A\*, the Bondi scalings may be applicable outside the circularization radius of  $\sim 10^3 - 10^4 r_g$  (Cuadra et al. 2006), although this need not be true in the presence of magnetic fields or significant heat conduction (e.g., Igumenshchev & Narayan 2002; Johnson & Quataert 2007).

In contrast to the Bondi model, analytic models (Blandford & Begelman 1999; Quataert & Gruzinov 2000) and numerical simulations (Stone & Pringle 2001; Hawley, Balbus, & Stone 2001) of rotating radiatively inefficient accretion flows find  $p \sim 1$ , which corresponds to an accretion rate which decreases with radius as  $\dot{M} \propto r$ . In this case,  $dRM/d\ln r \propto r^{-1/4}$ . There is thus a significant contribution to RM from both small and large radii in the flow.

### 3. Numerical Results

#### 3.1. The Fiducial Simulation

As a fiducial case, we focus on the simulation with  $r_0 = 200r_g$ , a resolution of  $120 \times 88$  grid points, and with initial magnetic field unit vectors shown in Figure 1 (“configuration A”). The resolution in the innermost portions of the torus is such that the fastest growing MRI mode is resolved with about 5 grid points. Different resolutions,  $r_0$ , and initial magnetic field configurations are discussed later. Several properties for these simulations are summarized in Tables 1 & 2.

The shear in the initial torus converts radial magnetic field into azimuthal field. Simultaneously, the MRI amplifies the magnetic field. The dynamical timescales are shortest at small radii in the torus, where mass flows in as it loses angular momentum because of magnetic stresses. Figure 2 shows the mass accretion rate through  $r_{\min} = 2r_g$  as a function of time for this simulation. A quasi-steady turbulent accretion flow is established after 5 orbital periods at  $100 r_g$  (note that 1 orbital pe-

riod at  $100 r_g$  is  $\approx 3 (M_{\text{BH}}/4 \times 10^6 M_\odot)$  days). The accretion rate is, however, highly variable and can sometimes change by an order of magnitude on short time scales (of order the orbital timescales at small radii). This is somewhat reminiscent of the flaring activity seen in Sgr A\* (Baganoff et al. 2001; Genzel et al. 2003).

Figure 3 shows contour plots of the density and plasma  $\beta$  (the ratio of thermal to magnetic pressure) for the fiducial simulation, averaged over the quasi-steady turbulent state. Poloidal magnetic field unit vectors are superimposed on the density contours. Figure 3 shows that a high density, moderately thin disk with very high  $\beta$  is formed in the equatorial plane of the initial torus (similar to Figures 4 & 5 of SP01). The accretion flow in the turbulent state is influenced by the initial magnetic field configuration. Shearing of the initially radial magnetic field above and below the equator leads to a strong toroidal field in the corona which compresses the midplane into a dense moderately thin disk with high  $\beta$ . By contrast, both the density and  $\beta$  are quite small in the polar regions. The average magnetic field vectors shown in Figure 3 for the quasi-steady structure clearly maintain memory of the initial magnetic field configuration shown in Figure 1.

For our fiducial simulation, we use  $r_{\text{in}} = 10r_g$  and  $r_{\text{out}} = 120r_g$  as the inner and outer radii in the RM integral (eq. [1]). Figure 4 shows the time averaged  $RM_{\text{code}}$  and the standard deviation in time along different lines of sight. As expected, the RM is fairly symmetric about the midplane ( $\theta = 90^\circ$ ), except for the sign change which arises because of the initial field configuration. In contrast to  $\langle RM_{\text{code}} \rangle$ , the standard deviation peaks at the midplane and is significantly smaller near the pole. This is because the midplane is highly turbulent, while the polar regions contain a comparatively stable magnetic field. Figure 4 shows that the time variability in RM is larger than the average RM except within  $\approx 40^\circ$  of the poles.

As discussed in §2.2, a more apt comparison to observations is the Faraday rotation averaged over a finite source size, rather than along individual lines of sight. Figure 5 shows  $RM_{\text{code}}$  and the standard deviation as a function of  $\theta$  for  $D_{\text{beam}} = 10r_g$ . The results in Figure 5 are reasonably similar to the single angle results shown in Figure 4. The primary difference is that the fluc-

tuations in the RM in the midplane ( $\theta \sim 90^\circ$ ) are smaller by a factor of  $\sim 3$  because the beam averages over the spatial fluctuations in the turbulent disk. Nonetheless, the temporal fluctuations implied by Figure 5 are still sufficiently large that one would only expect a relatively steady RM for polar viewing angles. Note also that both  $\langle RM_{\text{code}} \rangle$  and its standard deviation do not depend that strongly on viewing angle  $\theta$ ; there is roughly a factor of  $\sim 5$  variation from pole to equator.

To quantify which radii dominate the contribution to the RM, it is useful to consider the logarithmic derivative  $dRM/d\ln r$ . Figure 6 shows  $dRM_{\text{code}}/d\ln r$  as a function of radius for an equatorial and  $\theta = 30^\circ$  beam with  $D_{\text{beam}} = 10r_g$ ; both the time average and standard deviation are shown.  $dRM_{\text{code}}/d\ln r$  fluctuates with radius in Figure 6 because we are taking a numerical derivative with a finite number of grid points in the beam. For the equatorial line of sight, both the time average (solid) and standard deviation (dot-dashed) of RM receive roughly equal contributions from all radii. This is consistent with the analytic scalings reviewed in §2.3. In addition, as already shown in Figure 5, the standard deviation is significantly larger than the time averaged RM for the equatorial beam, i.e., the RM is highly variable in the turbulent disk. By contrast, the  $\theta = 30^\circ$  beam has a  $\langle dRM_{\text{code}}/d\ln r \rangle$  (dashed) which is somewhat larger than the standard deviation (dotted).

For the polar line of sight through the corona of the disk, both  $\langle dRM_{\text{code}}/d\ln r \rangle$  and the standard deviation are dominated by small radii, in contrast to the equatorial lines of sight where all radii contribute significantly. The solid line superimposed on the numerical results in Figure 6 is a  $dRM_{\text{code}}/d\ln r \sim r^{-3/2}$  power law which does a reasonable job at fitting the numerical results.

Spatial variations in the amount of Faraday rotation across a finite-sized source can lead to depolarization of the source at wavelengths where  $\sigma_\perp(RM)\lambda^2 \gtrsim 1$ , where  $\sigma_\perp(RM)$  is the spatial standard deviation in RM across the source (“beam depolarization”; e.g., Burn 1966; Tribble 1991). To quantify this effect in our simulations, Figure 7 shows the beam averaged RM and  $\sigma_\perp(RM)$  as a function of time for  $\theta = 30^\circ$  and  $\theta = 90^\circ$ , taking  $D_{\text{beam}} = 10r_g$  and  $r_{\text{in}} = 10r_g$ ;  $\sigma_\perp(RM)$  is calculated by subdividing the  $10r_g$  wide beam into smaller beams. The time aver-

age value of  $\sigma_\perp(RM)/RM$  is  $\approx 10$  for  $\theta = 90^\circ$  and  $\approx 0.3$  for  $\theta = 30^\circ$ ; it is somewhat smaller for  $\theta < 30^\circ$ . For a larger value of  $r_{\text{in}} = 25r_g$  we find similar results for  $\theta = 90^\circ$  but find that the spatial variation in RM is somewhat larger at  $\theta = 30^\circ$ , with an average value of  $\sigma_\perp(RM)/RM \approx 0.9$ .

### 3.2. Effects of the Initial Magnetic Field Configuration

The results of the previous section show that several properties of the quasi-steady turbulent state in the accretion flow at late times are determined by the initial magnetic field threading the torus at the beginning of the simulation. In particular, the antisymmetry of RM about the equatorial plane (Fig. 5) and the relatively coherent magnetic field at polar angles (Fig. 3 and 5) are all signatures of the initial magnetic field structure shown in Figure 1. It is thus important to understand which of our results on the disk structure and the RM are robust to changes in the initial field configuration. Figure 8 shows the initial magnetic field unit vectors for the alternative initial field configuration we considered (“configuration B”; see the Appendix for the vector potential used to generate this field). In this case, the initial field is symmetric about the midplane and has a net radial magnetic field in the equatorial plane of the initial torus;  $\beta$  is initially  $\approx 10 - 30$  in the equatorial plane and  $\approx 100 - 10^3$  in the rest of the torus. Note that although the initial magnetic field is still confined to the torus, the field lines are not along the constant density contours (as they were in Fig. 1). In addition, the vertical scale of the magnetic field structure in configuration B is roughly half that of configuration A. Thus we need a higher resolution to resolve the dynamics of the MRI. We take the simulation with  $r_0 = 100r_g$  and a resolution of  $240 \times 176$  as the standard case for initial field configuration B. For this case the fastest growing MRI modes at  $120r_g$  (just above and below the equator) are resolved with about 8 grid points.

Figure 9 shows 2-D contour plots of the density and plasma  $\beta$  averaged over the turbulent steady state. The resulting disk is significantly thicker compared to that arising from field configuration A (see Fig. 3). In addition, the disk scale-height is roughly proportional to the radius as in analytic models of radiatively inefficient accretion flows.

Unlike configuration A where  $\beta \gg 1$  in the disk midplane, here  $\beta \sim 1$  in the turbulent disk. The average field structure in the quasi-steady state is again reminiscent of the initial field configuration with radially inward field lines in the equatorial plane. The thick disk is surrounded by a relatively high  $\beta$  turbulent region, with fairly low  $\beta$  plasma at the poles. In general, the density and magnetic field structure of the disk is rather different from that in Figure 3. This is also shown in Figures 10 & 11, which compare the density and  $\beta$  within  $12^\circ$  of the equatorial plane averaged over the turbulent state as a function of radius for the two different initial magnetic configurations. The magnetic field is stronger for configuration B, and  $\beta$  decreases with radius, likely because of flux freezing of the initial radial field. In addition, the density does not increase significantly inwards for configuration B. These differences in the steady state disk structure appear to be a direct consequence of the different initial magnetic field structure. Although a net inward radial field may not always be a realistic initial condition, part of the motivation for choosing this particular configuration was to understand whether the RM in the bulk of the disk showed significant time variability even in the extreme case of a coherent initial radial field.

Figure 12 shows the time averaged RM and the standard deviation with  $D_{\text{beam}} = 10r_g$  as a function of angle  $\theta$ . The RM is roughly symmetric about the equatorial plane and peaks at  $\theta \approx 90^\circ$ . This is a direct consequence of the initial magnetic field configuration, which is symmetric about the equator and has a net radial field in the equator of the torus. Figure 12 also shows that the time variations in RM are larger than  $\langle RM_{\text{code}} \rangle$  at essentially all angles. This is in contrast to the results for initial magnetic field configuration A shown in Figure 5 where  $\langle RM_{\text{code}} \rangle$  is larger than the fluctuations near pole. In the present case, the polar viewing angles are relatively turbulent and show magnetic field reversals in time. Interestingly, although  $\langle RM_{\text{code}} \rangle$  is non-zero in the equator, the fluctuations due to the turbulence are still sufficiently large that one would expect significant time variability and sign changes in RM for equatorial viewing angles.

The time averaged  $dRM_{\text{code}}/d \ln r$  and its standard deviation for equatorial and  $\theta = 30^\circ$  beams

with  $D_{\text{beam}} = 10r_g$  are shown as a function of radius in Figure 13. For the polar viewing angle, the smallest radii dominate the contribution to both  $\langle RM_{\text{code}} \rangle$  and its time variation, with  $dRM_{\text{code}}/d \ln r \propto r^{-1}$  being a reasonable fit. For the equatorial beam, however, both  $\langle RM_{\text{code}} \rangle$  and the standard deviation are dominated by the largest radii. This is in contrast to the results shown in Figure 6 for initial field configuration A, and in contrast to the simple analytic estimates from §2.3. The significant contribution of large radii to  $\langle RM_{\text{code}} \rangle$  in the equatorial plane is in part because the density in the equator increases with radius for initial field configuration B (see Fig. 10).

### 3.3. Convergence Studies

To test whether our results depend on the resolution of the simulation, we carried out simulations at a number of different resolutions:  $60 \times 44$ ,  $120 \times 88$ , and  $240 \times 176$  for initial configuration A (see Table 1), and  $120 \times 88$  and  $240 \times 176$  for initial field configuration B (see Table 2). The density and magnetic field structure in the turbulent state is similar for all resolutions with initial configuration A. However, configuration B requires a higher resolution to resolve the dynamics of the MRI. As a result, the lower resolution simulation ( $120 \times 88$ ) has a smaller  $\langle \dot{M} \rangle$  than the higher resolution simulation ( $240 \times 176$ ).

As a dimensionless measure of whether our results for the RM have converged, we define the coherence parameter (Coh) as

$$\text{Coh} = \frac{\int_{r_{\text{in}}}^{r_{\text{out}}} n \mathbf{B} \cdot d\mathbf{l}}{\int_{r_{\text{in}}}^{r_{\text{out}}} n |\mathbf{B} \cdot d\mathbf{l}|}. \quad (3)$$

Coh quantifies the effect of cancellation due to magnetic field reversals on the inferred RM. If the large-scale dynamics of the magnetic field is properly resolved, one would expect that simulations with different resolution would have statistically similar values for Coh.

Figure 14 shows the beam averaged Coh at  $\theta = 90^\circ$  (the equator) as a function of time for initial configuration A for three different resolutions and  $r_0 = 100r_g$ . All three resolutions show statistically similar variations about  $\langle RM \rangle \approx 0$  (corresponding to a similar average and standard deviation for RM; see Table 1). The time variability

ity is also quite similar, implying that the resolution does not appear to be significantly affecting our determination of RM.

Figure 15 shows the beam averaged Coh for initial configuration B as a function of time in the equatorial plane for  $r_0 = 100r_g$ . The results are similar for the two different resolutions ( $240 \times 176$  and  $120 \times 88$ ). RM is negative at most times because of the initial magnetic field. There are, however, times when RM is positive, consistent with the large standard deviation shown in Figure 12.

Ideally for Sgr A\*, one would like to simulate an initial torus at  $r_0 \sim 10^5 r_g$ , so that the inner disk extends to  $\sim 10^3 - 10^4 r_g$ , similar to the expected circularization radius in Sgr A\* (e.g., Cuadra et al. 2006). Because of finite computational resources, however, we can only simulate a smaller disk. We carried out simulations with  $r_0 = 40r_g$  ( $r_{\text{out}} = 30r_g$ ),  $100r_g$  ( $r_{\text{out}} = 60r_g$ ), &  $200r_g$  ( $r_{\text{out}} = 120r_g$ ) for both initial field configurations A & B (see Tables 1 & 2). Although there are some quantitative differences between the different simulations, the basic results for RM, its statistical fluctuations, and its variation with radius, do not seem to depend significantly on the radial extent  $r_0$ , particularly when comparing the  $r_0 = 100r_g$  and  $r_0 = 200r_g$  simulations (the  $r_0 = 40r_g$  simulation has comparatively little radial dynamic range). This provides some confidence in the numerically calculated properties of RM through the accretion flow.

#### 4. Discussion

We have carried out an analysis of Faraday rotation in global axisymmetric MHD simulations of radiatively inefficient accretion flows, motivated by the observations of linear polarization and Faraday rotation in Sgr A\*. We first summarize our primary results and then discuss their implications for models of Sgr A\*.

We find that the accretion disk structure, and in particular the rotation measure RM, retains memory of the initial magnetic field threading the plasma, even in the quasi-steady turbulent state at late times. Physically, this is equivalent to a dependence on the properties of the magnetic field in the plasma feeding the accretion flow at large radii (which is typically hard to determine observationally). We considered two very different ini-

tial magnetic field geometries, one with a net radial field in the equator of the initial torus and one without (see Figs. 8 [configuration B] & 1 [configuration A], respectively). In configuration B, the initial magnetic field unit vectors are symmetric about the equatorial plane, while in configuration A they are antisymmetric. The initial symmetry of the magnetic field remains in the RM as a function of viewing angle  $\theta$  determined in the turbulent accretion flow (compare Figs 5 & 12). In addition, the steady state density and  $\beta$  profiles are rather different for the two different initial magnetic field configurations (Figs. 10 & 11). Given the anti-dynamo theorem, it is not that surprising that the initial field structure imprints itself on the late-time disk structure. However, the similarity of previous 2D and 3D global disk simulations (Hawley, Balbus, & Stone 2001) suggests that this will also be true in three dimensional simulations. It remains to be seen if simulations of accretion disks in three dimensions lose memory of the initial conditions at very late times (many orbital periods at large radii), e.g., by undergoing an inverse cascade which generates a large-scale magnetic field, or if the dependence on the large-scale initial magnetic field that we see is a generic and robust feature of accretion disk dynamos.

In spite of the dependence on the initial magnetic field threading the plasma, we can draw a number of general conclusions about the properties of Faraday rotation through magnetized geometrically thick accretion flows.

1. The RM through the midplane of the turbulent disk is highly time dependent, with a typical dispersion in RM that is larger than the mean. This implies significant changes in the observed sign of the RM. We see sign changes in RM even for our initial condition with a net radial field in the midplane of the torus (configuration B; Fig. 12), an initial condition that was chosen to maximize the likelihood of a coherent sign of RM.
2. In the turbulent disk, RM receives roughly equal contributions from all radii (Fig. 6 & 13). Analytic scaling arguments for disk models that are consistent with the simulations also suggest a weak dependence of RM on radius, with small radii dominating by a modest amount ( $dRM/d \ln r \propto r^{-1/4}$ ; §2.3).



3. RM is only a modest function of viewing angle  $\theta$  (see Figs 5 & 12); the polar RM is typically  $\sim 0.2$  of the equatorial RM for  $r_{\text{in}} = 10r_g$ .
4. If there is a large scale magnetic field threading the plasma at large radii (our configuration A), the resulting corona of the disk also has a large-scale, stable magnetic field (see also SP01, De Villiers 2006). The RM viewed through the corona has a dispersion in time smaller than the mean (Fig. 5) and is thus relatively stable.
5. For polar viewing angles, the RM is primarily produced at small radii; most of the contribution will thus arise from near the radius at which the electrons become relativistic (Fig. 6 & 13).
6. There is significant spatial variation in the amount of Faraday rotation through a finite sized beam, with  $\sigma_{\perp}(RM) \sim 0.3 RM$  at polar viewing angles (Fig. 7). This spatial variation in RM can cause depolarization at frequencies for which  $\sigma_{\perp}(RM)\lambda^2 \gtrsim 1$ .

#### 4.1. Implications for Sgr A\*

Our results confirm previous analytic arguments (e.g., Agol 2000; Quataert & Gruzinov 2000) that models of Sgr A\* with  $\dot{M} \sim \dot{M}_{\text{Bondi}} \sim 10^{-5} M_{\odot} \text{ yr}^{-1}$  are ruled out by the measured Faraday rotation of  $RM \approx -6 \times 10^5 \text{ rad m}^{-2}$ . For an accretion rate of  $\dot{M} \sim \dot{M}_{\text{Bondi}}$ , the electrons must be marginally relativistic even at small radii in order to account for the low luminosity from Sgr A\*. In this case, our choice of  $r_{\text{in}} = 10r_g$  as the minimum radius for which Faraday rotation is produced (see eq. [1]) is quite reasonable (and even conservative; in the ADAF models of Narayan et al. 1998, the electron temperature never exceeds  $6 \times 10^9 \text{ K}$ ). Our simulations show that for nearly all viewing angles  $\theta$  and for both of the initial magnetic field configurations we considered, the dimensionless rotation measure  $RM_{\text{code}} \gtrsim 10^{-4}$  (Figs. 5 & 12). Even though the time averaged  $RM$  may be smaller than this for certain viewing angles (e.g., through the equator of the turbulent disk,  $\theta = 90^\circ$ , in Fig. 5), the time variations are significant and so  $RM_{\text{code}} \gtrsim 10^{-4}$  is a good indication of the typical

instantaneous RM that would be observed at any  $\theta$ . Using equation (2) to convert RM to real units, we find that  $\dot{M} \sim \dot{M}_{\text{Bondi}} \sim 10^{-5} M_{\odot} \text{ yr}^{-1}$  would imply  $RM \sim 10^{10} \text{ rad m}^{-2}$ , over 4 orders of magnitude larger than the observed value. Thus such models are strongly ruled out.

We now discuss two scenarios that can account for the observed  $RM \approx -6 \times 10^5 \text{ rad m}^{-2}$  and its apparent stability over the past  $\approx 7$  years. These scenarios each identify the RM-producing region with a key radial scale in the inflowing plasma.

First, our simulations show that if there is a relatively coherent magnetic field in the plasma at large radii (configuration A) and if the observer's line of sight is through the polar regions of the accretion flow ( $\theta \sim 30^\circ$ ), then the time variability of RM appears to be sufficiently small to account for stable sign of RM observed from Sgr A\* (Figs. 5 & 7). For these polar viewing angles, we find  $RM_{\text{code}} \approx 3 \times 10^{-4}$ , i.e.,  $RM \approx 4.5 \times 10^{10} (\dot{M}_{\text{in}}/10^{-5} M_{\odot} \text{ yr}^{-1})^{3/2} \text{ rad m}^{-2}$ . The RM at polar viewing angles is produced at small radii with  $dRM/d\ln r \propto r^{-\alpha}$  and  $\alpha \approx 1 - 1.5$ . Our simulations have  $r_{\text{in}} = 10r_g$ , but  $r_{\text{in}} \sim 100r_g$  is probably a more accurate estimate of the radius at which the electrons become non-relativistic in Sgr A\* (e.g., Sharma et al. 2007). Rescaling our simulations to a larger value of  $r_{\text{in}}$  yields  $RM \approx 10^9 (\dot{M}_{\text{in}}/10^{-5} M_{\odot} \text{ yr}^{-1})^{3/2} (r_{\text{in}}/100r_g)^{-1.5} \text{ rad m}^{-2}$  for  $\alpha = 1.5$ . Thus  $RM \approx 6 \times 10^5 \text{ rad m}^{-2}$  requires  $\dot{M}_{\text{in}} \approx 7 \times 10^{-8} (r_{\text{in}}/100r_g) M_{\odot} \text{ yr}^{-1}$ . For  $\alpha \approx 1$ , which is also reasonably consistent with our simulations, we find  $\dot{M}_{\text{in}} \approx 3 \times 10^{-8} (r_{\text{in}}/100r_g)^{2/3} M_{\odot} \text{ yr}^{-1}$ . Because  $r_{\text{in}}$  is only moderately well-constrained, this estimate of  $\dot{M}_{\text{in}}$  is probably uncertain by a factor of  $\sim 5$ . In spite of this uncertainty, our estimated  $\dot{M}_{\text{in}}$  required to account for the observed RM from Sgr A\* is comparable to previous analytic estimates (e.g., Marrone et al. 2007); it is also similar to the accretion rate estimate of Sharma et al. (2007), who inferred  $\dot{M}_{\text{in}}$  by calculating the radiative efficiency of the accreting plasma in Sgr A\*.

A second possible explanation for the observed RM from Sgr A\* is that it is not directly produced within the nearly Keplerian accretion flow at small radii, but is instead produced in the more slowly rotating plasma further from the black hole. Numerical simulations of accretion onto Sgr A\* from stellar winds located at  $\sim 10^5 - 10^6 r_g$  find that the

circularization radius of the flow is  $r_{\text{circ}} \sim 10^3 - 10^4 r_g$  (Cuadra et al. 2006). As discussed in §1, the RM produced by the observed thermal plasma at  $\sim 10^5 r_g$  is  $\approx 5000 \beta^{-1/2} \text{ rad m}^{-2}$ , where  $\beta$  is the ratio of the thermal energy density to the magnetic energy density, assuming a magnetic field with a significant radial component. If the Bondi solution is applicable outside  $r_{\text{circ}}$ , then  $dRM/d \ln r \propto r^{-7/4}$  and the RM produced by the roughly spherical inflowing plasma at large radii is  $RM \approx 10^7 \beta^{-3/2} (r_{\text{circ}}/10^3 r_g)^{-7/4} \text{ rad m}^{-2}$  (roughly independent of viewing angle). Circularization radii in the range expected can thus plausibly account for the observed RM from Sgr A\* if  $\beta \sim 1$ . Note that in this interpretation,  $\dot{M}_{\text{in}}$  must be  $\ll 7 \times 10^{-8} (r_{\text{in}}/100 r_g) M_{\odot} \text{ yr}^{-1}$  (for  $\alpha = 1.5$ ) in order for the Faraday rotation from  $r \ll r_{\text{circ}}$  to be small compared to that from  $\sim r_{\text{circ}}$ .

This interpretation requires a significant magnetic field at large radii, with  $\beta \sim 1$ . Such a field could perhaps be produced by flux freezing of a moderately weaker field residing in the central parsec  $\sim 10^6 r_g$ . More interestingly, spherically inflowing plasma in Sgr A\* is unstable to the magnetothermal instability (MTI) which amplifies magnetic fields in a low collisionality plasma when the temperature increases in the direction of gravity (Balbus 2000; Parrish & Stone 2005). The MTI preferentially operates at large radii in spherical accretion flows ( $\gtrsim 10^4 r_g$ ) because only there is the growth time short compared to the inflow time (e.g., Fig. 9 of Johnson & Quataert 2007). In a hydrostatic atmosphere with a fixed temperature difference between the top and bottom of the atmosphere, the MTI leads to a significant radial heat flux, magnetic field amplification, and field lines combed out in the radial direction (Parrish & Stone 2005). Johnson & Quataert (2007) argued that the heat flux associated with the MTI could modify the dynamics of spherical accretion at large radii, which would in turn alter  $dRM/d \ln r$  from the Bondi estimate above. Thus the efficacy of the MTI for producing the observed Faraday rotation from Sgr A\* needs to be evaluated in a self-consistent dynamical calculation.

The above interpretations of the Faraday rotation observed towards Sgr A\*, namely that it arises either from  $r_{\text{in}} \sim 100 r_g$  or  $r_{\text{circ}} \sim 10^4 r_g$ , can probably be distinguished by careful monitoring of the variability of RM, which should be present at

some level even in the absence of sign changes. In the  $r_{\text{in}}$  scenario one would expect variability on  $\sim$  hour-day timescales, while in the  $r_{\text{circ}}$  scenario, the variability would occur on much longer timescales, of order years. The apparent lack of significant RM variability to date (e.g., Bower et al. 2005; Marrone et al. 2007) may favor models in which the RM is produced at large radii  $\sim r_{\text{circ}}$ .

If radii  $\sim r_{\text{in}}$  contribute significantly to the observed RM, our simulations can also be used to quantify whether Faraday depolarization can explain the rapid drop in the polarization fraction observed at frequencies below a few 100 GHz (e.g., Bower et al. 2003; Marrone et al. 2006). Beam depolarization becomes important when  $\sigma_{\perp}(RM)\lambda^2 \gtrsim 1$ , where  $\sigma_{\perp}$  is the variation in RM across a finite-sized source (e.g., Burn 1966; Tribble 1991). Our simulations have  $\sigma_{\perp}(RM) \sim 0.1 - 1 RM$  for polar viewing angles, with the exact value depending on  $\theta$  and  $r_{\text{in}}$  (Fig. 7 and §3.1). Taking  $\sigma_{\perp}(RM) \approx 0.3 RM$  as a fiducial value, and using the observed RM of  $\approx 6 \times 10^5 \text{ rad m}^{-2}$ , this implies that emission below  $\approx 150 \text{ GHz}$  should be depolarized by beam depolarization, in reasonable agreement with observations. Because of the variations in  $\sigma_{\perp}$  in our current simulations with  $r_{\text{in}}$  and  $\theta$ , higher resolution simulations and simulations with  $r_{\text{in}} \sim 100 r_g$  will be required to more accurately determine the frequency below which beam depolarization becomes important. Figure 7 also shows that  $\sigma_{\perp}(RM)$  increases by a factor of  $\sim 10$  at certain times. This could give rise to a significant decrease in the observed polarization similar to that observed by Marrone et al. (2006) at 340 GHz.

In a recent paper, Ballantyne et al. (2007) argue that the “normal” Faraday rotation we have focused on in this paper may not be the dominant process shaping the observed polarization from Sgr A\*. Instead, they argue that because the plasma is relativistic over a large range of radii, the normal modes of the plasma are elliptical. In this case, the observed polarization probes the properties of the normal modes in the radio-emitting region, which in turn depends on the magnetic field and temperature structure of the plasma. However, current observations of Sgr A\* show that the change in the polarization angle from 100-400 GHz is consistent with the  $\lambda^2$  variation expected for normal Faraday rotation (e.g., Macquart et al.

2006; Marrone et al. 2007). This strongly suggests that non-relativistic plasma at large radii is modifying the intrinsic polarization of the source via Faraday rotation. It also requires that the intrinsic polarization angle of the source is relatively independent of frequency. Finally, we note that the source size is inferred to decrease significantly at the frequencies for which linear polarization is detected, from  $\approx 10r_g$  at 100 GHz to  $\approx 2r_g$  at 400 GHz (Bower et al. 2004). Thus different frequencies sample different plasma along the line of sight propagating to observers on Earth. To produce the same RM independent of frequency requires that the source size at all frequencies is much smaller than the size of the region where the RM is produced (so that there is no variation in RM with frequency). This criterion is reasonably satisfied for both of the interpretations of the Faraday rotation from Sgr A\* considered above.

Throughout this paper we have focused on the implications of our results for Sgr A\* because of the wealth of observational data available. We anticipate, however, that as observational techniques improve our simulations will prove useful for interpreting the physical conditions in the vicinity of other accreting black holes.

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### A. Vector potential for configuration B

To define the vector potential for initial field configuration B, we set up a new cylindrical coordinate system whose origin is on the equator of our standard spherical coordinate system at a distance of  $1.3r_0$  (roughly the mid-point of the initial torus), and whose x and y axes are aligned with the local  $-\theta$  and  $-r$  axes of the spherical coordinate system, respectively. The vector potential for initial field configuration B is then given by

$$\begin{aligned} A &= 0.005 \cos\left(\frac{\pi}{2} \frac{r_{cy}}{r_m}\right) \cos(\phi) && \text{for } r_{cy} < r_m, \\ &= 0 && \text{otherwise,} \end{aligned} \tag{A1}$$

where  $r_{cy}$  is the radius in the cylindrical coordinate system,  $r_m = 0.5r_0$ , and  $\phi$  is the angle with respect to the x- axis in the cylindrical coordinate system. The vector potential is located at cell corners and can be differenced to get the magnetic field unit vectors shown in Figure 8.

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TABLE 1  
RM FOR INITIAL FIELD CONFIGURATION A

$r_0$	Resolution	$\langle \dot{M}_{\text{code}} \rangle$	$\langle \text{Coh} \rangle^{\text{a}}$	$\sigma(\text{Coh})^{\text{a}}$	$\langle RM_{\text{code}} \rangle^{\text{a}}$	$\sigma(RM_{\text{code}})^{\text{a}}$	$\langle \text{Coh} \rangle^{\text{b}}$	$\sigma(\text{Coh})^{\text{b}}$	$\langle RM_{\text{code}} \rangle^{\text{b}}$	$\sigma(RM_{\text{code}})^{\text{b}}$
200 $r_g^{\text{c}}$	$120 \times 88$	0.0025	-0.1	0.22	$-1.1 \times 10^{-4}$	$4.1 \times 10^{-4}$	0.85	0.13	$2.8 \times 10^{-4}$	$10^{-4}$
200 $r_g$	$60 \times 44$	0.002	0.04	0.16	$8.3 \times 10^{-5}$	$2.7 \times 10^{-4}$	0.95	0.088	$2.1 \times 10^{-4}$	$10^{-4}$
100 $r_g$	$120 \times 88$	0.0029	-0.05	0.14	$-1.6 \times 10^{-4}$	$3.9 \times 10^{-4}$	0.83	0.14	$1.6 \times 10^{-4}$	$5.7 \times 10^{-5}$
100 $r_g$	$60 \times 44$	0.0022	-0.04	0.17	$-8.5 \times 10^{-5}$	$2.6 \times 10^{-4}$	0.93	0.12	$2.3 \times 10^{-4}$	$6.8 \times 10^{-5}$
100 $r_g$	$240 \times 176$	0.0033	0.04	0.18	$8 \times 10^{-5}$	$3.4 \times 10^{-4}$	0.63	0.26	$1.4 \times 10^{-4}$	$9 \times 10^{-5}$
40 $r_g$	$120 \times 88$	0.0031	-0.06	0.12	$-2.2 \times 10^{-4}$	$3.7 \times 10^{-4}$	0.66	0.24	$9.5 \times 10^{-5}$	$5.7 \times 10^{-5}$
40 $r_g$	$60 \times 44$	0.0028	-0.21	0.17	$-4.2 \times 10^{-4}$	$3.1 \times 10^{-4}$	0.84	0.15	$1.9 \times 10^{-4}$	$8.1 \times 10^{-5}$
40 $r_g$	$240 \times 176$	0.0039	-0.03	0.1	$-1.4 \times 10^{-4}$	$4 \times 10^{-4}$	0.69	0.22	$8.5 \times 10^{-5}$	$5.8 \times 10^{-5}$

NOTE.— $\langle \rangle$  ( $\sigma$ ) represents a time average (temporal standard deviation) in the turbulent state. The time average is taken over the quasi-steady turbulent state which lasts from  $\approx 3 - 10$  orbits at  $r_0$  depending on the simulation. All RM results are also averaged over a  $10r_g$  beam.

<sup>a</sup> $\theta = 90^\circ$

<sup>b</sup> $\theta = 30^\circ$

<sup>c</sup>The fiducial run.

TABLE 2  
RM FOR INITIAL FIELD CONFIGURATION B

$r_0$	Resolution	$\langle \dot{M}_{\text{code}} \rangle$	$\langle \text{Coh} \rangle^{\text{a}}$	$\sigma(\text{Coh})^{\text{a}}$	$\langle RM_{\text{code}} \rangle^{\text{a}}$	$\sigma(RM_{\text{code}})^{\text{a}}$	$\langle \text{Coh} \rangle^{\text{b}}$	$\sigma(\text{Coh})^{\text{b}}$	$\langle RM_{\text{code}} \rangle^{\text{b}}$	$\sigma(RM_{\text{code}})^{\text{b}}$
200 $r_g$	120 $\times$ 88	0.0016	-0.17	0.27	$-5.9 \times 10^{-4}$	$9.1 \times 10^{-4}$	0.45	0.29	$7.7 \times 10^{-5}$	$9 \times 10^{-5}$
100 $r_g$	120 $\times$ 88	0.0031	-0.1	0.25	$-3.4 \times 10^{-4}$	$8.8 \times 10^{-4}$	0.64	0.25	$6.9 \times 10^{-5}$	$4.6 \times 10^{-5}$
100 $r_g$	240 $\times$ 176	0.0061	-0.2	0.21	$-3.4 \times 10^{-4}$	$3.8 \times 10^{-4}$	0.15	0.45	$3.2 \times 10^{-5}$	$10^{-4}$
40 $r_g$	120 $\times$ 88	0.0059	-0.29	0.17	-0.0012	$7.2 \times 10^{-4}$	0.47	0.33	$1.2 \times 10^{-5}$	$1.9 \times 10^{-5}$
40 $r_g$	240 $\times$ 176	0.0095	-0.01	0.12	$-7.1 \times 10^{-6}$	$4.6 \times 10^{-4}$	0.13	0.35	$10^{-5}$	$2.6 \times 10^{-5}$

NOTE.— $\langle \rangle$  ( $\sigma$ ) represents a time average (temporal standard deviation) in the turbulent state. The time average is taken over the quasi-steady turbulent state which lasts from  $\approx 3 - 10$  orbits at  $r_0$  depending on the simulation. All RM results are also averaged over a  $10r_g$  beam.

<sup>a</sup> $\theta = 90^\circ$

<sup>b</sup> $\theta = 30^\circ$

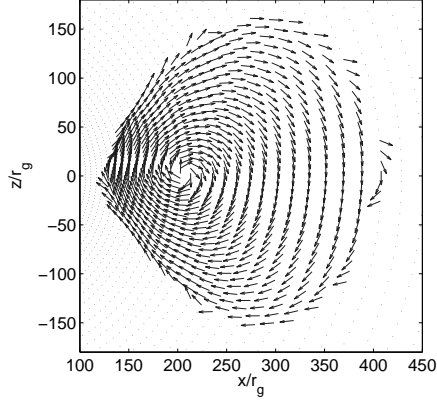


Fig. 1.— Magnetic field unit vectors for initial field configuration A. The initial magnetic field is confined within the dense torus and is along constant density contours.

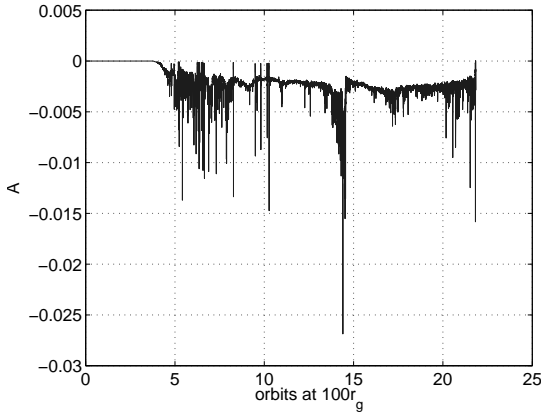


Fig. 2.— The mass accretion rate through the inner boundary ( $r_{\min} = 2r_g$ ) for the fiducial run. There is large variability in the accretion rate on timescales comparable to the orbital period at small radii.

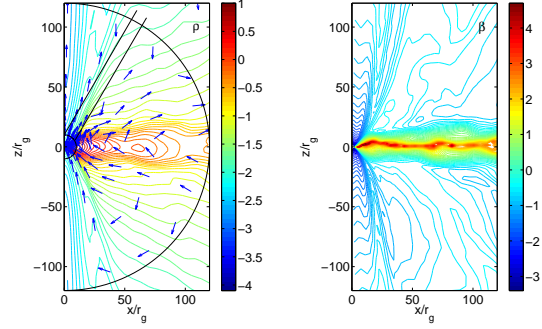


Fig. 3.— Contour plots of density ( $\rho$ , left) and  $\beta = 8\pi p/B^2$  (right) in the quasi-steady turbulent state for the fiducial run (averaged from 6 to 20 orbits at  $100r_g$ ). Arrows indicate the average magnetic field direction. Dense high  $\beta$  plasma is confined to a relatively thin disk; the density decreases significantly towards the poles, where the plasma is strongly magnetized (low  $\beta$ ). The inner ( $r_{\text{in}} = 10r_g$ ) and outer ( $r_{\text{out}} = 120r_g$ ) radii used in our rotation measure integrals are shown. Also shown is a fiducial “beam” with a diameter of  $D_{\text{beam}} = 10r_g$  at  $\theta = 30^\circ$  used to calculate the rotation measure for a finite sized source.

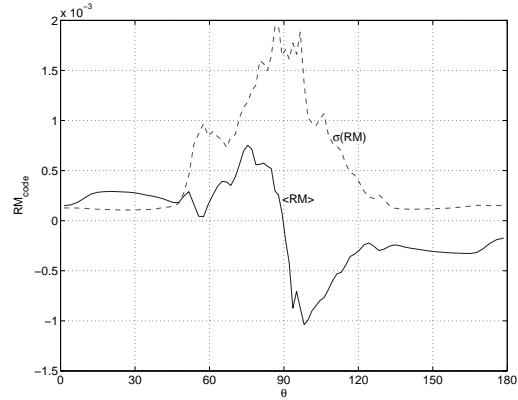


Fig. 4.— Time averaged RM (solid line) and standard deviation in time (dashed line) along individual lines of sight ( $\theta = \text{constant}$ ) in the turbulent steady state for the fiducial run ( $r_{\text{in}} = 10r_g$ ,  $r_{\text{out}} = 120r_g$ ). Temporal fluctuations in RM in the turbulent disk ( $\sim 10^\circ$  about the midplane) are much larger than the average RM. Only in the polar regions ( $\sim 40^\circ$  about the poles) are the fluctuations in RM smaller than the mean.



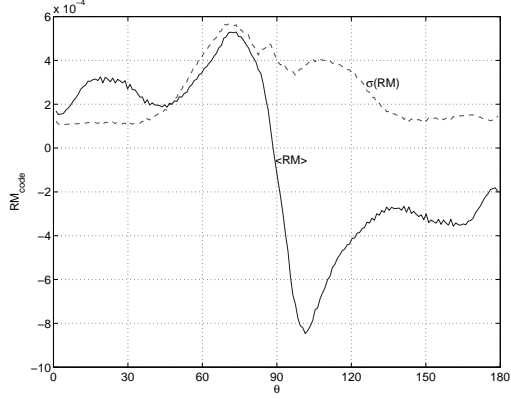


Fig. 5.— Time averaged RM (solid line) and standard deviation in time (dashed line) in a  $10r_g$  diameter beam as a function of viewing angle for the fiducial run ( $r_{\text{in}} = 10r_g$ ,  $r_{\text{out}} = 120r_g$ ). Fluctuations near the midplane of the disk ( $\theta \sim 90^\circ$ ) are somewhat smaller than for the individual lines of sight shown in Figure 4. Nonetheless, only near the poles ( $\theta \sim 30^\circ$ ) are the fluctuations smaller than the mean.

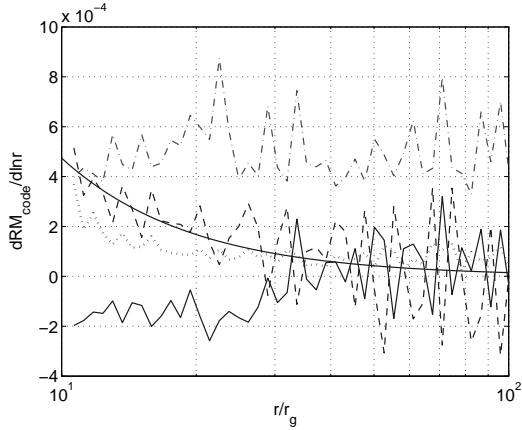


Fig. 6.—  $dRM_{\text{code}}/d\ln r$  for a  $10r_g$  beam for the fiducial simulation. Time average (solid line) and standard deviation in time (dot-dashed line) for an equatorial viewing angle and time average (dashed line) and standard deviation in time (dotted line) for a  $30^\circ$  viewing angle are shown. Also shown is an  $r^{-3/2}$  fit to  $\langle dRM_{\text{code}}/d\ln r \rangle$  for  $\theta = 30^\circ$ . The curves are oscillatory in part because  $dRM_{\text{code}}/d\ln r$  requires taking a numerical derivative.

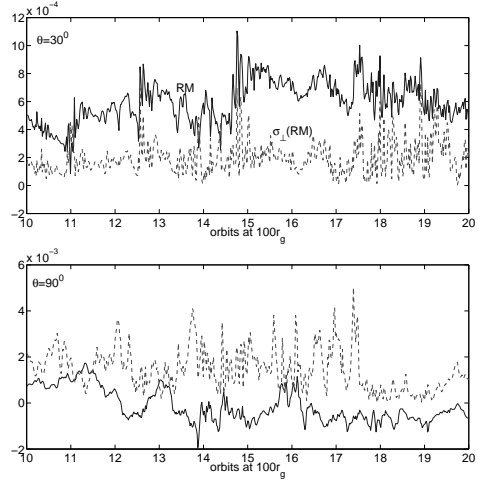


Fig. 7.— Beam-averaged ( $D_{\text{beam}} = 10r_g$ ) RM (solid line) and standard deviation across the beam  $\sigma_{\perp}(RM)$  (dashed line) as a function of time for the fiducial simulation; one orbital period at  $100 r_g$  is  $\approx 3 (M_{\text{BH}}/4 \times 10^6 M_{\odot})$  days. Note that  $\sigma_{\perp}(RM)$  in this Figure is a standard deviation *across the beam*, rather than a standard deviation *in time*, which is shown in the remaining Figures. *Top*:  $\theta = 30^\circ$ ; *Bottom*:  $\theta = 90^\circ$ . The emission is beam depolarized when  $\sigma_{\perp}(RM)\lambda^2 \gtrsim 1$ . Because the RM at polar viewing angles is primarily produced at small radii (Fig. 6), the characteristic timescale for RM and  $\sigma_{\perp}(RM)$  variability scales roughly as  $r_{\text{in}}^{3/2}$ ; these calculations have  $r_{\text{in}} = 10r_g$ .

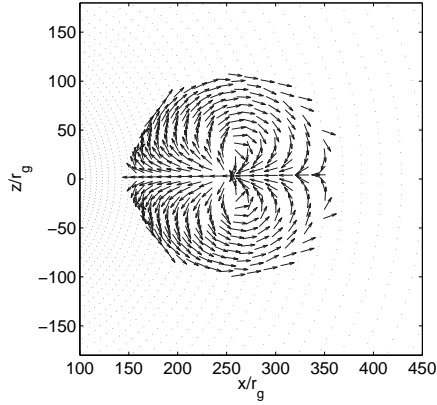


Fig. 8.— Initial magnetic field unit vectors for field configuration B. In contrast to the initial magnetic field for the fiducial simulations (Fig. 1), in this case the field is symmetric about the equatorial plane and there is a net inward radial field in the equator.

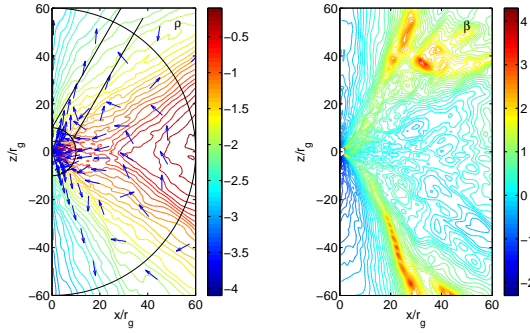


Fig. 9.— Contour plots of density ( $\rho$ , left) and  $\beta = 8\pi p/B^2$  (right) in the quasi-steady turbulent state for initial field configuration B with  $r_0 = 100r_g$  and resolution  $240 \times 176$  (averaged from 1.5 to 3.5 orbits at  $100r_g$ ). Arrows indicate the average magnetic field direction. The disk is less dense, more strongly magnetized ( $\beta \sim 1$ ), and much thicker than for the fiducial simulation (see Fig. 3). The magnetic field in the corona is less ordered than for the fiducial simulation, though there is still very low  $\beta$  plasma at the poles. The inner ( $r_{\text{in}} = 10r_g$ ) and outer ( $r_{\text{out}} = 60r_g$ ) radii used in our rotation measure integrals are shown. Also shown is a fiducial “beam” with a diameter of  $D_{\text{beam}} = 10r_g$  at  $\theta = 30^\circ$  used to calculate the rotation measure for a finite sized source.

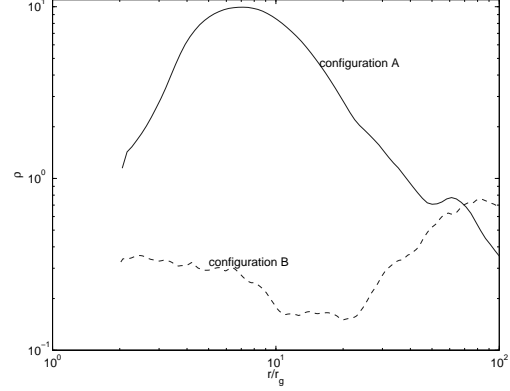


Fig. 10.— Density in the equatorial plane as a function of radius, averaged over the turbulent state for initial field configuration A (solid; Fig. 1) and B (dashed; Fig. 8). The density is averaged over a  $12^\circ$  wedge centered on the midplane.

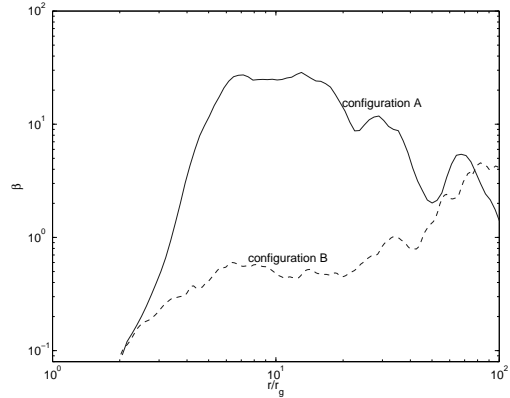


Fig. 11.— Plasma  $\beta$  in the equatorial plane as a function of radius, averaged over the turbulent state for initial field configuration A (solid; Fig. 1) and B (dashed; Fig. 8);  $\beta$  is averaged over a  $12^\circ$  wedge centered on the midplane. The plasma  $\beta$  is significantly smaller for configuration B, likely because of flux freezing of the initial radial field.

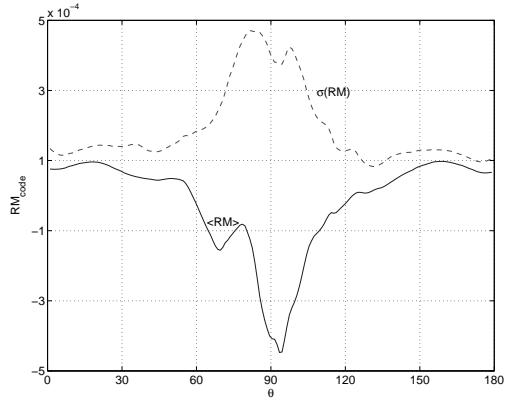


Fig. 12.— Time averaged RM (solid line) and standard deviation in time (dashed line) in a  $10r_g$  diameter beam as a function of viewing angle for initial field configuration B (for resolution  $240 \times 176$ ,  $r_{\text{in}} = 10r_g$ ,  $r_{\text{out}} = 60r_g$ , and  $r_0 = 100r_g$ ). The fluctuations in RM exceed the mean at essentially all angles.

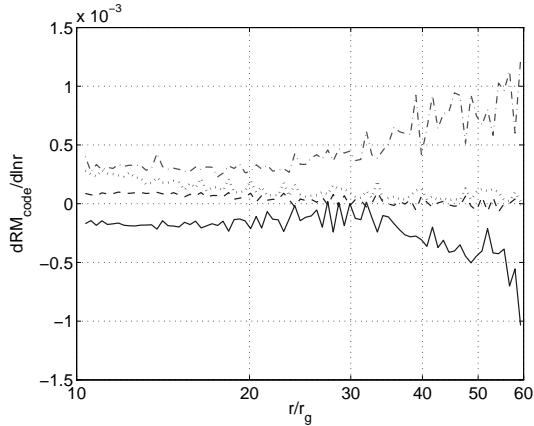


Fig. 13.—  $dRM_{\text{code}}/d \ln r$  for a  $10r_g$  beam for initial field configuration B (for resolution  $240 \times 176$ ,  $r_{\text{in}} = 10r_g$ ,  $r_{\text{out}} = 60r_g$ , and  $r_0 = 100r_g$ ). Time averaged (solid line) and standard deviation in time (dot-dashed line) for an equatorial viewing angle and time average (dashed line) and standard deviation in time (dotted line) for a  $30^\circ$  viewing angle are shown.  $dRM_{\text{code}}/d \ln r$  decreases with increasing radius for the  $30^\circ$  viewing angle while it increases with increasing radius for the equatorial viewing angle. The curves are oscillatory in part because  $dM_{\text{code}}/d \ln r$  requires taking a numerical derivative.

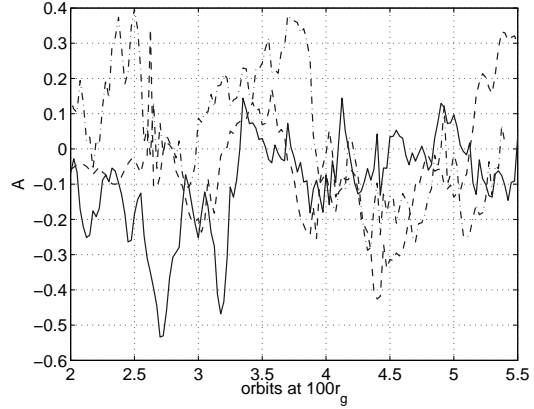


Fig. 14.— The effect of magnetic field reversals on the rotation measure is quantified by the Coherence (see eq. [3]). Here Coh is shown as a function of time for three different resolutions for the fiducial run:  $120 \times 88$  (solid line),  $60 \times 44$  (dashed line), and  $240 \times 176$  (dot-dashed line). An equatorial viewing angle and a  $10r_g$  beam is assumed for these calculations.

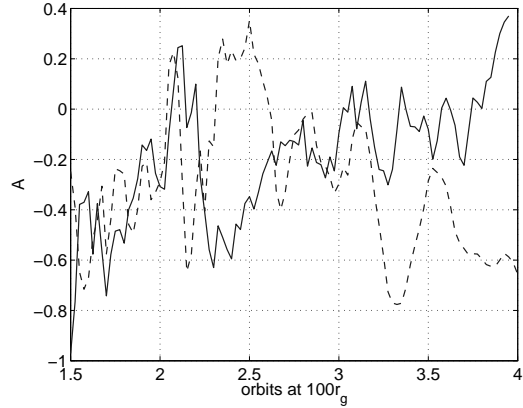


Fig. 15.— Coherence (see eq. [3]) as a function of time for initial field configuration B at two resolutions (with  $r_{\text{in}} = 10r_g$ ,  $r_{\text{out}} = 60r_g$ , and  $r_0 = 100r_g$ ):  $120 \times 88$  (solid line) and  $240 \times 176$  (dashed line). An equatorial viewing angle and a  $10r_g$  beam is assumed for these calculations. Note that the Coh is positive at some times, demonstrating that RM in the midplane can change sign even though there is an initially radial field threading the equator of the torus.